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Investigation of boundary layer development in Low Pressure Turbine Cascades with unsteady incoming wakes – Effect of Reynolds Number

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ABSTRACT

This paper presents an investigation of the impact of the use of a flat plate instead of a cascade on suction surface boundary layer developments of a low pressure turbine under the influence of incoming wakes.

The boundary layer parameters measured in a low speed cascade rig with a moving bar mechanism were compared with those estimated by flat plate correlations formulated by Coull and Hodson (2010) for low Reynolds number at Re_c = 56,000 (Kodama et al., 2012). It was found that empirical modeling of unsteady boundary layer using a flat plate would tend to raise a boundary layer growth on the suction surface with increasing wake reduced frequency.

The present study compares the boundary layer developments on the suction surface of the cascade with those calculated by the flat plate correlations for higher Reynolds number at Re_c = 100,000 and investigate the effects of Reynolds number on the difference in suction surface boundary layer growth between the use of a flat plate and a cascade.

NOMENCLATURE

C	Chord
C_X	Axial Chord
C_p	Static Pressure Coefficient
DF	Diffusion Factor (Eq. (9))
f_r	S_0 -based Wake Reduced Frequency (Eq. (7))
H	Boundary Layer Shape Factor
P_{01}	Inlet Total Pressure
Re_C	Chord-based Reynolds Number
Re_{S_0}	S_0 -based Reynolds Number
Re_θ	θ -based Reynolds Number
S	Surface Distance from the Leading Edge
S_0	Overall Suction Surface Length
U	Local Freestream Velocity
$\Delta U^*/\Delta S^*$	Non-dimensional Deceleration Rate (Eq. (6))
ν	Kinematic Viscisity
θ	Momentum Thickness

Subscripts

$peak$	Peak Velocity Location
sep	Separation Location
TE	Suction Surface Trailing Edge

INTRODUCTION

Because of a high aspect ratio design for low pressure (LP) turbines of a high bypass ratio turbo fan engine, profile loss is the main loss caused by the blades in a LP turbine. The profile loss is generated in the development of airfoil boundary layers and the magnitude of the profile loss highly depends on the process of boundary layer transition and separation at LP turbine operating conditions. Due to the turbulence associated with the wakes shed by upstream blade rows, the transition process has an unsteady nature. It has been shown that unsteady wakes has a large influence on transition process (e.g. Halstead et al., 1995). Therefore many efforts have been conducted to obtain a better understanding of unsteady interaction between incoming wakes and surface boundary layer.

In several experimental efforts, moving-bar mechanisms have been used to simulate wakes. Schulte and Hodson (1996) examined the influences of wake-passing frequency and wake strength on the profile loss using a moving bar mechanism and a low speed cascade. Curtis et al. (1997) and Howell et al. (2001) used the same cascade wind tunnel with the moving-bar mechanism to investigate high lift LP turbine blade profiles changing a velocity profile on the suction surface. The variations in the velocity distribution were achieved by fitting an adjustable flap to the trailing edge of the airfoil adjacent to the one under investigation in the datum cascade. In addition to the flap, a number of inserts were also placed on the pressure side of the passage.

Coull et al. (2010) investigated a wide range of suction surface velocity distributions and clarified the effects of unsteady wakes on the suction surface boundary layer loss. However the experiments were performed using moving bars upstream of a flat plate. In this experiments, symmetric, contoured liners above and below the flat plate were used to impose the various velocity distributions on the flat plate surface.

The wake behavior as it passes through a blade row is quite different from that passes through a free stream. The wake becomes highly distorted and stretched because the part adjacent to the suction surface convects more rapidly than that adjacent to the pressure surface. The velocity deficit of the center of the wake is reduced by this inviscid effect (e.g. Denton, 1993). Therefore the effect of unsteady wakes on the growth of suction surface boundary layer in a blade row is considered to be different from that on the growth of boundary layer on a flat plate.

Kodama et al. (2012) investigated the impact of the use of a flat plate instead of a cascade on the suction surface boundary layer development of a low pressure turbine under the influence of incoming wakes. In the investigation, the boundary layer parameters measured in a low speed cascade rig with a moving bar mechanism (Funazaki et al., 2006, 2010) were compared with those estimated by the correlations, which were formulated by Coull and Hodson (2010) using the measurements in a flat plate rig with a moving bar mechanism. It was found that the difference between the use of a flat plate and a cascade had a significant impact on the growth of suction surface boundary layer under the influence of incoming wakes. However, the comparisons were made only at a low Reynolds number Re_c = 56,000.

This paper aims at investigating the effects of Reynolds number on the difference in suction surface boundary layer growth between the use of a flat plate and a cascade. The boundary layer measurements performed by Funazaki et al. (2006, 2010) using a low speed cascade are compared with the empirical correlations formulated by Coull and Hodson (2010) at higher Reynolds number Re_c = 100,000.

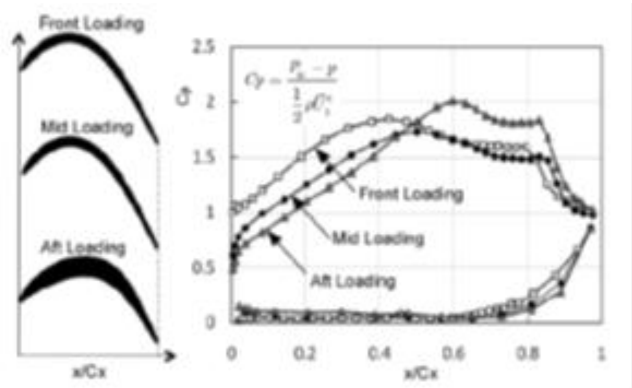

Fig. 1 Cross sections and static pressure distributions of three test airfoils (Funazaki et al., 2010)

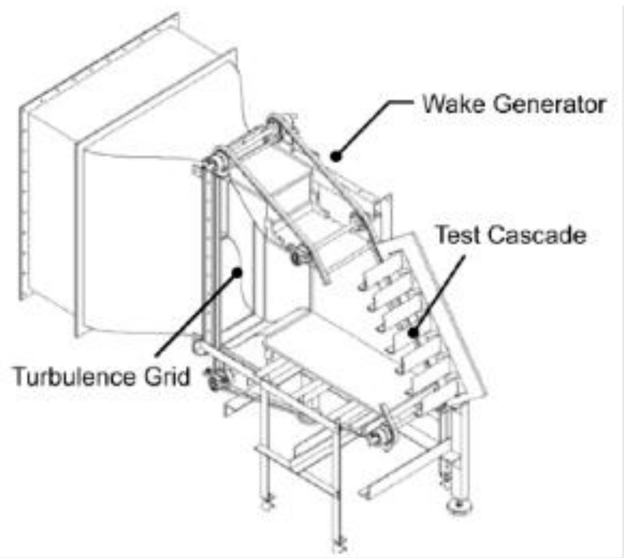

Fig. 2 Test section of low speed moving bar cascade facility (Funazaki et al., 2010)

CASCADE MEASUREMENT

The boundary layer measurements for this study were performed using a low speed cascade facility at Iwate University (Funazaki et al., 2010).

CASCADE DETAILS

Three kinds of cascades with different airfoil geometries were used in the boundary layer measurements. These airfoil geometries were designed with the same axial chord length (100 mm), inlet flow angle (47 deg) and exit flow angle (-60 deg), but with different chord-wise loading distributions. Consequently, the velocity distributions on the suction surfaces varied as shown in Figure 1.

EXPERIMENTAL SETUP

A schematic of the test section is shown in Figure 2. This consists of the linear cascade of six or seven blades, where the number of blades depended on the type of the airfoil geometry used in the experiments, and a moving bar mechanism that allows bars to be traversed upstream of the leading edge of the cascade. Cylindrical bars of 3mm diameter (3% axial chord of the cascade) fitted between two timing belts were driven by an inverter-controlled induction motor so that moving wakes were generated at a plane 1.15 axial chords upstream of the cascade.

INSTRUMENTS

In the center part of the cascade, there were two brass blades in which pressure tappings were instrumented to measure static pressure distributions over the suction and pressure surfaces. The inlet total pressure P_{01} was measured using a miniature Pitot tube at midspan location 0.72 axial chords upstream from the leading edge of the instrumented blade on the pressure side.

A single hot-wire probe was used for boundary layer measurements. The axial location of the measurements extended from 50% of axial chord to the blade trailing edge and, in the normal direction to the blade suction surface, the measurements covered from 0.2% of axial chord (0.2mm) to 10% of axial chord (10mm). A PC-controlled traversing unit, equipped downstream of the cascade with minimal blockage, enabled us to automatically position the probe along a normal line to the airfoil surface.

Uncertainty analysis showed that the inlet velocity error was ± 1.7%, the static pressure coefficient was ± 3.5% and the hot-wire probe measurements was ± 2%.

FLAT PLATE CORRELATION

For a given velocity distribution measured on a suction surface of the cascade, boundary layer parameters are predicted by using empirical correlations formulated by Coull and Hodson (2010). According to their accuracy analysis of the correlation, the overall standard deviation in the momentum thickness at trailing edge was less than 3% with bias error of -0.3% relative to their flat plate measurements (Coull and Hodson, 2010). Therefore, in this study, the correlated values of boundary layer parameters are dealt with as the values that would be measured on a flat plate.

MOMENTUM THICKNESS AT LAMINAR SEPARATION

A calculation of laminar momentum thickness up to a separation point is performed using the method of Thwaites (1949) that achieved excellent accuracy at Re_c = 56,000 (Kodama et al., 2012). The momentum thickness at laminar separation is expressed by:

$$\theta_{sep}^2 = \frac{0.45\nu}{U_{sep}^6}\int_0^{S_{sep}} U^5 dS \qquad (1)$$

In the present calculation, S_{sep} is predicted by Stratford method (Stratford, 1954) as the accuracy was verified by the previous work (Kodama et al., 2012).

TRAILING EDGE MOMENTUM THICKNESS

A correlation for the trailing edge momentum thickness is composed of three terms $[\theta_{TE}]_{BUBBLE}$, $[\theta_{TE}]_{TURB}$, and $[\theta_{TE}]_{WAKE}$, which represent the contribution of separation bubble, the contribution of turbulent boundary layer and the contribution of unsteady interaction, respectively.

$$[\theta_{TE}] = [\theta_{TE}]_{BUBBLE} + [\theta_{TE}]_{TURB} + [\theta_{TE}]_{WAKE} \tag{2}$$

Where

$$[\theta_{TE}]_{BUBBLE} = \theta_{sep}\left(6.69\times10^{6}\right) \times \left(\mathrm{Re}_{\theta_{sep}}\right)^{\left(-4.28+2.05\frac{\Delta U^*}{\Delta S^*}-0.161 f_r\right)} \tag{3}$$

$$[\theta_{TE}]_{TURB} = \theta_{sep}\left(\mathrm{Re}_{\theta_{sep}}\right)^{0.322} \times \left(1-\frac{S_{sep}}{S_0}\right)\left\{0.166+3.13\left(\frac{\Delta U^*}{\Delta S^*}\right)\right\} \tag{4}$$

$$[\theta_{TE}]_{WAKE} = S_0\left(7.06\times10^{-4}\right)\left(f_r\right) \tag{5}$$

Here

$$\frac{\Delta U^*}{\Delta S^*} = \frac{\left(U_{sep}-U_{TE}\right)/U_{sep}}{\left(S_0-S_{sep}\right)/S_0} \tag{6}$$

$$f_r = \left(\frac{f_{wake}S_0}{U_{TE}}\right) \tag{7}$$

As the momentum thickness at suction surface trailing edge dominates the profile loss (Coull and Hodson, 2010), it can be said from the above equations that the important parameters which determine a profile loss are the momentum thickness at laminar separation θ_{sep}, the non-dimensional deceleration rate $\Delta U^*/\Delta S^*$ and the wake reduced frequency f_r.

TRAILING EDGE DISPLACEMENT THICKNESS

The displacement thickness at suction surface trailing edge also contributes to the total profile loss. In the process of displacement thickness estimation, a shape factor at trailing edge is first predicted by the following equation:

$$H_{TE} = 1.545+\left(2.57\times10^{9}\right) \times \left(\mathrm{Re}_{S_0}\right)^{\left(-1.93DF+0.619\frac{\Delta U^*}{\Delta S^*}-0.085 f_r-1.77\right)} \tag{8}$$

Here diffusion factor DF is defined by:

$$DF = \frac{\left(U_{peak}-U_{TE}\right)}{U_{TE}} \tag{9}$$

Then the displacement thickness is estimated using the momentum thickness from Equation (2).

RESULTS AND DISCUSSION

COMPARISON OF VELOCITY DISTRIBUTION PARAMETERS

The suction surface velocity distributions for three different airfoils measured in a cascade rig test (see Figure 1) are used to estimate the correlated boundary layer growth that would be measured in a flat plate rig test. Table 1 compares the parameters regarding the shape of design velocity distributions between the flat plate measurements (Coull et al., 2010) and the cascade measurements (Funazaki et al., 2010). As seen in Table 1, DF of Aft Loading cascade and $\%S_0$ of Front Loading cascade are out of the design space of the velocity distributions for the flat plate measurements. However, as seen in Equation (2)-(5), the most important parameter is $\Delta U^*/\Delta S^*$ and its values used in the cascade tests are in a range of the parameter values for the flat plate correlation.

Table 1 Summary of parameters regarding design velocity distribution

Experimental Model	Velocity Distribution	Diffusion Factor DF Eq. (9)	Peak Velocity Location $\%S_0$	$\Delta U^*/\Delta S^*$ Eq. (6)
Flat Plate ([1],[6])	A	40%	42	0.54
	B	40%	52	0.64
	C	28%	42	0.45
	D	28%	52	0.52
	E	28%	62	0.66
	F	16%	42	0.28
	G	16%	52	0.33
	H	16%	62	0.43
Cascade [9]	Front Loading	36%	35	0.45
	Mid Loading	31%	41	0.42
	Aft Loading	41%	49	0.56

COMPARISON OF TEST CONDITIONS

In the flat plate rig tests (Coull et al., 2010), the Reynolds number Re_c was varied between 50,000 and 220,000. The test data treated in the current study were acquired at higher Reynolds number (Re_c = 100,000) than the Reynolds number (Re_c = 56,000) treated in the previous study (Kodama et al., 2012)

Table 2 Reduced frequency equivalent to Strauhal Number

Velocity Distribution	Strauhal Number St, Eq. (10)	Reduced Frequency f_r, Eq. (7)
Front Loading	0	0
Front Loading	0.4	0.38
Front Loading	0.8	0.74
Front Loading	1.2	1.13
Mid Loading	0	0
Mid Loading	0.4	0.36
Aft Loading	0	0
Aft Loading	0.4	0.36
Aft Loading	0.8	0.71
Aft Loading	1.2	1.08

The bar wake passing frequency was characterized by Strouhal number in the cascade measurements (Funazaki et al., 2010).

$$St = \left(\frac{f_{wake} C}{U_{IN}} \right) \tag{10}$$

In Table 2, the wake passing frequencies defined by reduced frequency (Equation (7)) are listed. The moving bar tests with a flat plate were performed at three different reduced frequencies up to f_r =1.26 (Coull et al., 2010). Therefore the difference in unsteady interaction between suction surface boundary layer and incoming wakes between flat plate and cascade measurements can be evaluated in almost the same range of the wake passing frequency.

All the cascade test data used in this study were acquired without turbulence grids. In the operation without moving bars, the inlet turbulence was 0.8%. This is smaller than the freestream turbulence level of 3% set in the flat plate rig tests (Coull et al., 2010). However, it was confirmed in the cascade measurements that the freestream turbulence levels were kept almost the same in a wide range of the wake passing frequency. So it can be considered that the difference in the freestream turbulence may not affect the variation of difference between the flat plate and the cascade measurements with wake passing frequency.

COMPARISON OF MOMENTUM THICKNESS AT LAMINAR SEPARATION

Figure 3 compares the momentum thickness calculated by the Thwaites method up to laminar separation with the boundary layer measurements for Aft Loading airfoil at f_r =0.36 at Re_c = 100,000. The comparison shows an excellent agreement as well as the comparison at Re_c = 56,000.

Percent difference in momentum thickness at laminar separation is defined by:

$$\frac{\left(\theta_{sep}\right)_{calculated} - \left(\theta_{sep}\right)_{measured}}{\left(\theta_{sep}\right)_{measured}} \times 100 \tag{11}$$

In Figure 4, the differences in θ_{sep} are plotted for all of the cascade measurements at Re_c = 100,000 (blue lines) and those at Re_c = 56,000 (red lines) are also plotted for comparison. The horizontal axis shows a wake reduced frequency. Almost all of the calculated values are within ±10% of the measured values. The effects of the Reynolds number on the predictions cannot be seen.

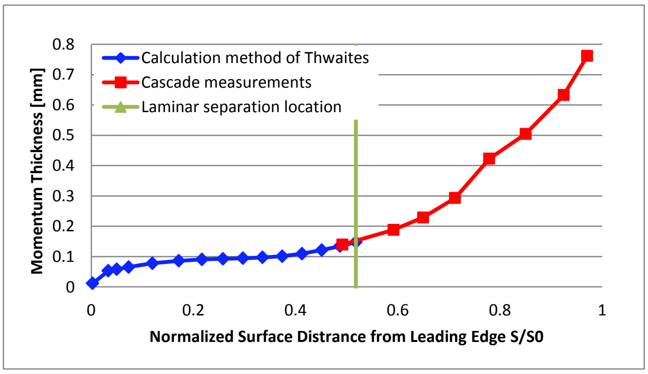

Fig. 3 Comparison of momentum thickness growth between cascade measurements and predictions by the method of Thwaites for Aft Loading (f_r =0.36, Re_c = 100,000)

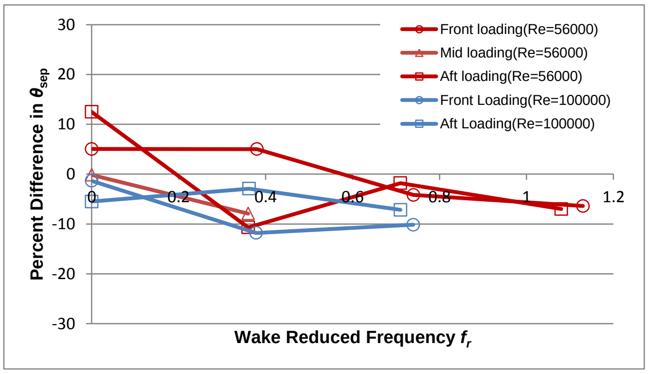

Fig. 4 Comparison of percent difference in momentum thickness at laminar separation between cascade measurements and predictions by the method of Thwaites between the two Reynolds numbers

COMPARISON OF DISPLACEMENT THICKNESS AT LAMINAR SEPARATION

While the displacement thickness at laminar separation is not taken into account in the estimation of a suction surface boundary layer loss, laminar boundary layer displacement thickness at pressure surface trailing edge contributes to the profile loss. Coull and Hodson (2010) recommended White method (White, 1991) for predicting a laminar boundary layer shape factor in the process of the displacement thickness estimation.

$$\begin{aligned} H &= 2.0 + 4.14z - 83.5z^2 + 854z^3 \\ &\quad - 3337z^4 + 4576z^5 \end{aligned} \tag{12}$$

Where:

$$z = \left(0.25 - \lambda_\theta\right) \tag{13}$$

$$\lambda_\theta = \frac{\theta^2}{\nu} \frac{dU}{dS} \tag{14}$$

In the previous work (Kodama et al., 2012), Holstein and Bohlen method (Schlichting, 1979) for predicting the laminar boundary layer shape factor was also evaluated. It was found that, although the White method predicted slightly larger shape factors than those predicted by the Holstein and Bohlen method, there was little significant difference between the two methods. However the resultant displacement thicknesses calculated by using Equation (1) were much larger than the measurements at Re_c = 56,000 for both methods.

In Figure 7, the calculated displacement thickness up to laminar separation by the White method is compared with the boundary layer measurements for Aft Loading airfoil at f_r =0.36 at Re_c = 100,000. The qualitative and quantitative features are very similar to those in the comparison at Re_c = 56,000. That is, the growth rate of calculated displacement thickness is similar to the measured growth rate around laminar separation location. However, there is a large discrepancy in its level between the predictions and the measurements.

Figure 8 shows the percent difference in the displacement thickness at laminar separation relative to the measurement against wake reduced frequency for all of the cascades. In the Figure, blue lines represent the results obtained at Re_c = 100,000 and red lines represent those at Re_c = 56,000. The results of Aft Loading airfoil at Re_c = 100,000 lie in the range between -20% and -40% where all the results at Re_c = 56,000 exit. On the other hand, the results of Front Loading airfoil at Re_c = 100,000 lie in the range between 0% and -10%. This error spread in a wide range is considered to be attributed to accuracy of the prediction of shape factor.

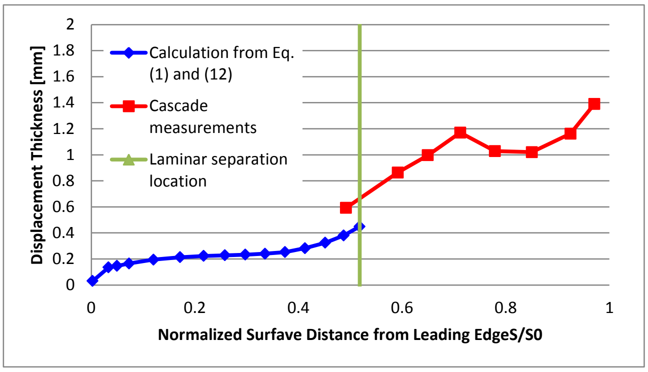

Fig. 7 Comparison of displacement thickness growth between cascade measurements and predictions using Eq. (1) and Eq. (12) for Aft Loading (f_r =0.36, Re_c = 100,000)

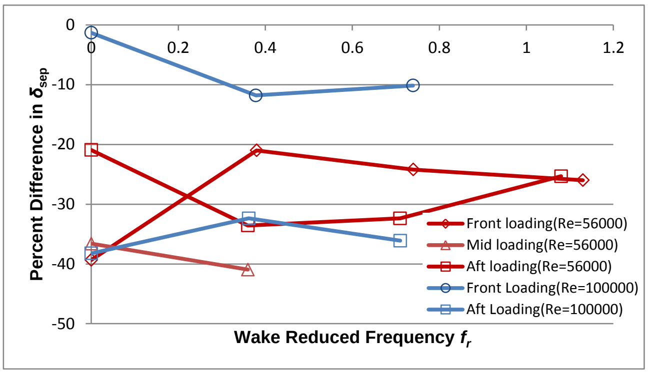

Fig. 8 Comparison of percent difference in displacement thickness at laminar separation between cascade measurements and predictions by the method of white between the two Reynolds numbers

COMPARISON OF MOMENTUM THICKNESS AT TRAILING EDGE

In Figure 9, the momentum thickness estimated by the Coull and Hodson correlation (Equation (2)) is compared with the boundary layer measurement for Aft Loading airfoil at f_r =0.36 at Re_c = 100,000. The measured momentum thickness shows a rapid growth from 0.5 *S/S0* to the trailing edge. The Coull and Hodson correlation predicts the trailing edge momentum thickness quite well.

Figure 10 shows the percent difference in the momentum thickness at trailing edge between the measurements and the correlations against wake reduced frequency. Again the results at Re_c = 100,000 (blue lines) are compared with the results at Re_c = 56,000 (red lines) in this figure. It was observed for the low Reynolds number at Re_c = 56,000 that the correlation tends to under-predict the momentum thickness at low wake reduced frequency and over-predict it at high wake reduced frequency. This indicated that a flat plate rig test would measure smaller momentum thickness at low wake reduced frequency and larger momentum thickness at high wake reduced frequency for the same free stream velocity distribution. This tendency can be also seen in the correlated results for the high Reynolds number at Re_c = 100,000. However the slopes of the increasing percent differences are much gradual compared with those for the low Reynolds number at Re_c = 56,000. This indicates that the difference in empirical modeling between the use of a flat plate and the use of a cascade might impact on the growth of suction surface boundary layer under the influence of Reynolds number as well as the influence of incoming wakes.

As shown in Figure 4, the prediction of momentum thickness up to laminar boundary layer separation is not almost influenced by incoming wakes and Reynolds number. Therefore it can be

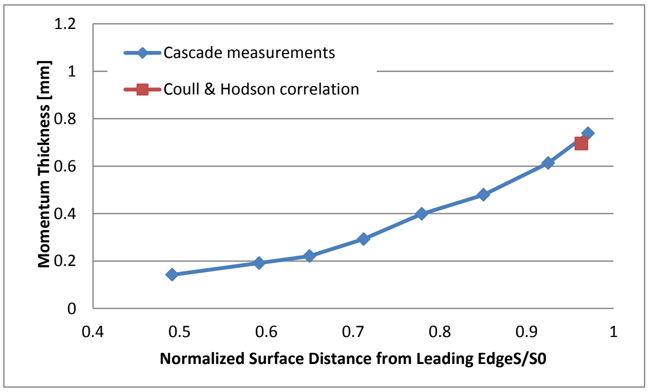

Fig. 9 Comparison of momentum thickness between cascade measurements and Coull and Hodson correlation for Aft Loading (f_r =0.36, Re_c = 100,000)

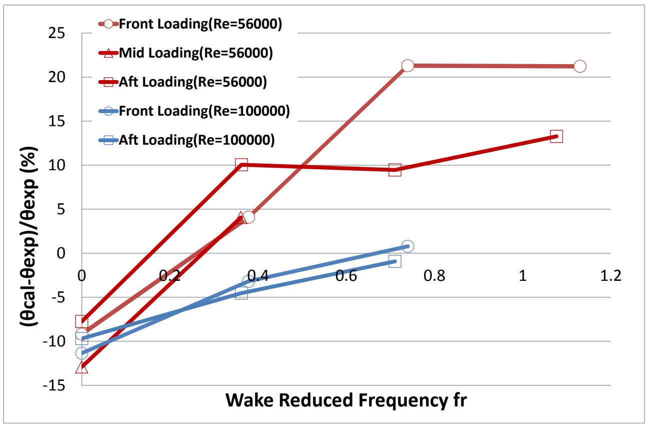

Fig. 10 Comparison of percent difference in momentum thickness at trailing edge between cascade measurements and Coull and Hodson correlation

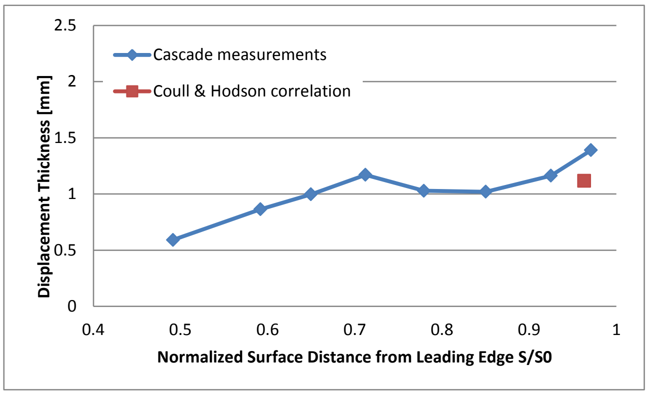

Fig. 11 Comparison of displacement thickness between cascade measurements and Coull and Hodson correlation for Aft Loading (f_r =0.36, Re_c = 100,000)

considered that the influences of incoming wakes and Reynolds number on the growth of suction surface boundary layer may appear after the laminar boundary layer separation. It should be also noted that all the correlated values exist in a small range between -7% and -13% of the measured values at f_r =0 in Figure 10. This indicates that the influence of Reynolds number may appear only when a suction surface boundary layer is subjected to incoming wakes. In the Coull and Hodson correlations, the wake interaction term $[\theta_{TE}]_{WAKE}$ is independent of Reynolds number (see Equation 5). Further investigation is necessary to understand the contribution of Reynolds number to the suction surface boundary layer growth.

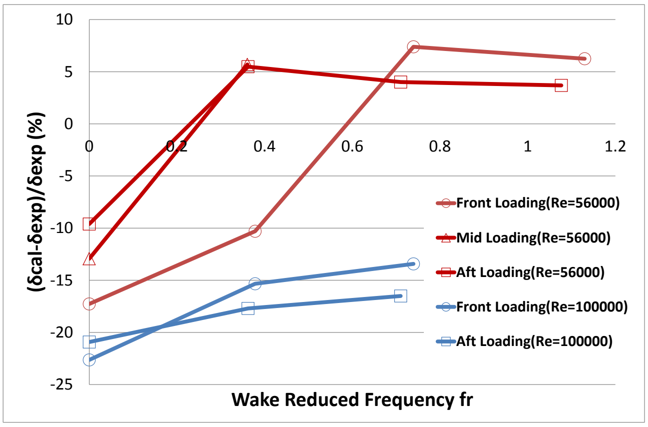

Fig. 12 Comparison of percent difference in displacement thickness at trailing edge between cascade measurements and Coull and Hodson correlation

COMPARISON OF MOMENTUM THICKNESS AT TRAILING EDGE

The displacement thickness at the trailing edge is calculated using correlated momentum thickness (Equation (2)) and correlated shape factor (Equation (8)). Figure 11 shows the correlated displacement thickness comparing with a plot of the measured displacement thickness over the suction surface. While there is a certain amount of discrepancy in the laminar boundary layer displacement thickness up to a separation point between the predictions and measurements (Fig. 7), the correlated displacement thickness at the trailing edge reasonably agrees with the measurements.

In Figure 12, the percent difference between the measured and the correlated displacement thickness at the trailing edge is plotted against wake reduced frequency for all of the cascade measurements at Re_c = 100,000 (blue lines) and at Re_c = 56,000 (red lines). The overall trends with wake reduced frequency are similar to those observed in Figure 10. It seems that a flat plate correlation would underestimate a displacement thickness at trailing edge for higher Reynolds number.

CONCLUSION

The impact of empirical modeling using a flat plate in experiments on growth of airfoil suction surface boundary layer in a low pressure turbine under the influence of incoming wakes has been discussed. The correlations which were formulated from flat plate test data with acceptable accuracy are used to estimate the boundary layer parameters for a given suction surface velocity distribution. The correlated parameters are compared as the parameters that would be measured in a flat plate rig test with those measured in a cascade rig test. The present study focuses on the influence of Reynolds number on the difference in the empirical modeling.

The tendency that a flat plate correlation tends to under-predict momentum thickness at low wake reduced frequency and over-predict it at high wake reduced frequency, which was seen at low Reynolds number, is also observed at high Reynolds number. However the slope of the variation with wake reduced frequency is much gradual at high Reynolds number compared with that at low Reynolds number. This suggests that the difference in empirical modeling between the use of a flat plate and the use of a cascade might impact on the growth of suction surface boundary layer under the influence of Reynolds number as well as the influence of incoming wakes.

Further investigation is necessary to understand the influence of Reynolds number on the difference in suction surface boundary layer growth between the use of a flat plate and a cascade.

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