Measurements of Heat Transfer around Turbine Blades Using Boundary Element Method and Uncertainty Analysis (Part I)

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Abstract

A numerical technique using Boundary Element Method(BEM) is proposed for the purpose of calculation of heat transfer coefficients on turbine blades, where constant elements or linear elements are used. In both cases thermal boundary conditions for BEM analysis are temperature distributions on the hot side blade surface and then heat transfer coefficients and coolant temperatures on the internal cooling-side boundaries, which are both experimental values. Detailed check on the numerical accuracy are made for some simple test cases and as a result it is found out that BEM using linear elements produces better results than BEM using constant elements. But when the latter is applied to a turbine blade case, the results almost agree with those of previous FEM analysis. But linear elements bring about a little difference from FEM. Due to rather conflicting results, there arise great necessities for further investigations.

Nomenclature

d = diameter of j-th inner cooling passage, [m]
G = Green function defined eq.(3)
h = heat transfer coefficient on inner surface, [W/m]
h = heat transfer coefficient on blade surface, [W/m]
k = thermal conductivity of fluid, [W/(m·K)]
N = element number on an outer boundary
N = element nimber on an inner boundary
n = normal vector, outwardly positive
P = Prandtl number
q = heat flux, [W/m^2]
R = Reynolds number
r = inner diameter, [m]
r = outer diameter, [m]
Γ_i = inner boundary
Γ_o outer boundary
δ = Dirac's delta function
θ = temperature, [K]
θ_i = temperature of inner surface, [K]
θ_o = temperature of outer surface, [K]
λ = thermal conductivity of blade material, [W/(m·K)]
ϕ = interpolating function defined by eq.(16)

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Subscript

c = cooling side
g gas(hot) side
j = number of cooling passage
o = outer side

1. Introduction

Designers of air-cooled turbine blades for aero-engines are nowadays confronted with two big difficulties. The first difficulty is that under the plateau of development in materials for hot section components, they must design the cooling configurations of blades which can realize high level of TIT(Turbine Inlet Temperature) with limited amount of cooling air. The another one is that they are requested to guarantee durabilities or specified lives of those blades in the very hostile environment where various influential factors on these characteristics are involved but always hard to be specified quantitatively. In both cases, it is clear that one of the key technologies is an accurate prediction of gas-side surface heat transfer.

As for the numerical predictions, althoughconsiderable amount of studies are made on those subjects and there're some examples of success to some degree [1-4], they are still far from reliable ones. It is because those techniques heavily depend on turbulence modelling with many empirical constants and correlations which derived from relatively simple test configurations and conditions. Therefore, in addition to the fundamental researches on the turbulence boundary layer, extensive and well-controlled measurements of blade surface heat transfer are strongly needed in order to accumulate database for direct comparison with numerical predictions.

Among a number of methods for measuring heat transfer around turbine blades[5-8], Turner's method[9] is a unique but reliable one which doesn't need any special measuring skills. The basic concept of his method is, as pointed out by Nealy et al[10], to use a turbine blade as a kind of fluxmeter. Heat flux distribution on the blade surface can be obtained by solving 2-D heat conduction equation in the blade material numerically with proper thermal boundary conditions obtained from experiments. After Turner himself used a variational method as a solver, Nealy et al[10], Witting et al[11] conducted extensive measurements of heat transfer distribution using Finite Element Method(FEM) instead. On the other hand, Nakata and Araki[12] proposed the use of Boundary Element Method(BEM). Their proposal seems to be reasonable and attractive one if temperature-dependence of the thermal conductivity of the blade material can be ignored, because only information which is neccesary to obtain the heat transfer is the blade surface heat flux distribution and that is what BEM can directly give in answer to the problem. One of the difficulties of Nakata Arakis'method is the specification of internal boundary conditions. They specified Dirichlet temperature condition on internal boundaries, which are obtained from thermocouples installed there, but in a practical sense such installation is not so easy, and even if it is possible, there may arise great errors, especially over large curvature regions.

This paper describes some modifications on the Nakata Arakis'BEM, where heat transfer coefficients on the internal circular boundaries are given instead of temperature distributions. As far as concerned, there're less experiences in application of BEM to this kind of problem, so that more works are still needed in order to accumulate informations especially about its accuracy, which will turn out to be great help in evaluating the final results, i.e. heat transfer. Therefore, in this paper as the first report of consecutive studies for measuring heat transfer on turbine blades, much efforts are concentrated on checking the accuracy of BEM using both simple test cases and turbine blade case.

2. Formulation

2.1 Basic equation

Suppose that the spanwise variation of blade metal temperature can be neglected, steady heat conduction equation becomes as follows:

$$\nabla^2\theta = 0 \ , \nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 \qquad (1)$$

While Brebbia[14] utilized the weighted residual methods in order to formulate integral equations from eq (1), in this paper rather classical Green's second identity is used for that purpose. The Green's identity can be written as:

$$\int_D (u\nabla^2 v - v\nabla^2 u)dx = \int_\Gamma (u\nabla v - v\nabla u)n ds, \qquad (2)$$

where normal vector n is positive if it is outwardly directed from the boundary. Considering that $u=\theta$ and $v=G$, where Green function G satisfies the following equation as:

$$\nabla^2 G(x,x') = \delta(x-x'), \ \text{:Dirac's delta function} \qquad (3)$$

we have temperature distribution in domain D as:

$$\theta(x) = \int_\Gamma (\theta\nabla G - G\nabla\theta)n ds. \qquad (4)$$

Eq.(3) can be easily solved and G turns out to be

$$G(x,x') = \ln(r)/2 \quad , \ r=|x-x'| \qquad (5)$$

When a point x is on the boundary of D, the following integral equation(5) can be obtained, taking extracircular integral path around that point with the infinitesimal radius.

$$-\theta(x)/2 + \int_\Gamma \theta\partial G/\partial n ds - \int_\Gamma G\partial\theta/\partial n ds. \qquad (6)$$

Even in the case of a multiple-connected domain shown in Fig.1, almost same equation can be derived.

$$-\theta(x)/2 + \int_{\Gamma_0} \theta\partial G/\partial n ds + \sum_j \int_{\Gamma_{ij}} \theta\partial G/\partial n ds = \int_{\Gamma_0} G\partial\theta/\partial n ds + \sum_j \int_{\Gamma_{ij}} G\partial\theta/\partial n ds. \qquad (7)$$

2.2 Numerical discretization

In the conventional BEM introduced by Brebbia[14], isoparametric elements are commonly used, where orders of interpolations of values(θ, $\partial\theta/\partial n$) (m) and its geometry(n) on each element are the same. Owing to remarkable progresses in BEM, we have a large choice of elements in terms of orders of interpolation and types (m=n:iso-,m>n:sub ,m<n:super-[15]). Although it is interesting to examine thoroughly what kind of elements can produce better results in terms of accuracy, it surely needs considerable time and efforts which might be beyond what this first paper can cover. Therefore in this paper simple and conventional constant and linear elements(panels) are used. In both cases boundaries are approximated by a number of straight lines. The other reason why these two ways of approximation are used is that from some experiences it is felt any higher order elements don't always give good results in spite of increasing cumbersomeness of codings.

Dividing the boundaries into elements, eq.(7) becomes

$$-\theta(x)/2 + \sum \int \theta\partial G/\partial n ds + \sum\sum \int \theta\partial G/\partial n ds = \sum \int G\partial\theta/\partial n ds + \sum\sum \int G\partial\theta/\partial n ds. \qquad (8)$$

Applying constant- or linear-element approximations to the above integral equation, it can be reduced to the following simultaneous linear equations.

$$-\theta_i/2 + \sum_j H_{ij}\theta_j = \sum_j G_{ij}q_j \qquad (9)$$

or

$$\sum_j H'_{ij}\theta_j = \sum_j G_{ij}q_j$$

where $q = \partial\theta/\partial n$.

Note that H_{ij} and G_{ij} in eq.(9) are written as follows, provided that all indices are difined like in Fig.2:

1) constant element

$$H_{ij} = \int \partial G/\partial n ds \qquad (10)$$

$$G_{ij} = \int G ds \qquad (11)$$

2) linear element

$$H_{ij} = h_{ij}^{(1)} + h_{ij}^{(2)} \qquad (12)$$

$$G_{ij} = g_{ij}^{(1)} + g_{ij}^{(2)} \qquad (13)$$

where

$$h = \int \Phi_1 \partial G/\partial n ds, \ h = \int \Phi_2 \partial G/\partial n ds \qquad (14)$$

$$g = \int \Phi_1 G ds \quad , \ g = \int \Phi_2 G ds \qquad (15)$$

$$\Phi_1 = (1-\xi)/2 \quad , \ \Phi_2 = (1+\xi)/2. \qquad (16)$$

It should be noted that diagonal elements in matrix H'_{ij} are known to be calculated by the following equation:

$$H'_{ii} = -\sum_{i \neq j} H'_{ij} \tag{17}$$

3. Application to heat transfer calculation

3.1 Heat transfer coefficient

Heat transfer coefficient h_g on the outer boundary is difined by

$$\begin{aligned} h_g &= \lambda(\partial\theta/\partial n)/(\theta_g-\theta) \\ &= -\lambda q/(\theta_g-\theta) \quad : \text{on } \Gamma_0 \end{aligned} \tag{18}$$

where λ is a thermal conductivity of a blade material and θ_g is surrounding gas temperature. It should be noted that λ and θ_g are assumed to be constant and also known.

3.2 Boundary conditions

It is obvious from eq.(18) that only information which is necessary to calculate h_g at a point on Γ_0 is the derivative of temperature(q) and it can be easily obtained by solving eq.(9) with proper thermal boundary conditions. Generally speaking, there're three types of boundary conditions, Dirichlet, Neumann and the mixed, and in this case outer boundary condition should be Dirichlet type, which means temperature distribution is specified on the boundary. On the other hand, as for the internal boundary condition, we might have a choice between Dirichlet type and mixed one. In the latter case heat transfer coefficients($h_{c,j}$)are specified and temperatures and those derivatives are enforced to satisfy the following relation

$$-\lambda(\partial\theta/\partial n) = h_{c,j}(\theta_{c,j}-\theta) \quad : \text{on } \Gamma_{i,j} \quad j=1,..,n \tag{19}$$

where $\theta_{c,j}$ is the cooling air temperature of j-th internal boundary.

York et al.(using FEM)[16],Nakata and Araki[14] specified temperature distributions on outer and inner boundaries to obtain surface heat transfer. Temperatures were obtained from thermocouples installed there, whereas Nakata and Araki only published results of numerical experiments. From the experimental point of view, however, it rather seems unpractical to use temperature distributions as boundary condition, because it isn't so easy to install sufficient number of thermocouples especially on the internal boundaries free from any significant effects on heat conduction inside the blade.

On the other hand, using turbine blades with inner circular cooling holes, Turner[9],Nealy et al[10] specified mixed boundary conditions as given in eq.(19) on the surfaces of cooling holes. Heat transfer coefficients on the surfaces($h_{c,j}$) were given from well-known correlations such as[17]

$$h = \frac{0.152\, Re^{0.9} Pr}{0.833[2.25\ln(0.144 Re^{0.9})+13.2Pr-5.8]} \frac{k}{d_j} \tag{20}$$

or simply

$$h = 0.022 Pr^{0.5} Re^{0.8} \tag{21}$$

Eq.(19),(20) are for the case of constant heat rate. In the case of constant surface temperature slight modification are made on eq.(20) and we have

$$h = 0.021 Pr^{0.5} Re^{0.8} \tag{22}$$

It should be noted that above correlations show fairly good agreements with experimental data, provided that

i) Surface of an inner passage is hydraulically smooth.

ii) turbulent flow inside the passage is fully developed or entrance effect is completely diminished.

iii) the ratio of inner surface temperature to fluid bulk temperature is not so large.

and

iv) effect of peripheral heat flux variation can be ignored

In spite of several assumptions included, this approach seems to be attractive one compared with that of Nakata and Araki because of its feasibility, so that eq.(19) are used as internal boundary condition in the following analysis.

4 Results

4.1 Check of accuracy

4.1.1 Effect of element number

To start things off, it is important to check accuracy of the codes developed this time. For that purpose we compared several numerical solutions by twotypes of BEM proposed in the previous section with analytical solutions. Because there are not so many examples whose exact solutions are known in explicit forms, we take advantage of rather simple cases of annular tube shown in Fig.3 which are heated on the outer surfaces and at the same time internally cooled. In these cases, taking account of the correspondence with applications to real bladecases, temperature distributions are specified on the outer surfaces and heat transfer coeffiecients and cooling air temperatures are specified on the inner surfaces.

First of all, effects of element numbers on accuracy are examined to the uniform heating case,in which its exact solution for heat flux on the outer surface is given by the following equation

$$\begin{aligned} \partial\theta/\partial n &= \partial\theta/\partial r \\ &= -\frac{\theta_0 - \theta_i}{r_2 \ln(r_2/r_1)} \end{aligned} \tag{23}$$

where

$$\theta_i = \frac{\theta_0 C + \theta_c h_c}{C+h_c}, \quad C = \frac{\lambda}{r_2 \ln(r_2/r_1)} \tag{24}$$

Fig.4 shows curves of relative errors by BEM of constant elements and linear elements versus number of elements, where

$$r_2 = 0.02, r_1 = 0.01, \theta_0 = 500, \theta_i = 300, \lambda = 16. \tag{25}$$

It should be noted that numbers of elements on outer and inner surfaces are fixed to be the same in this case. Both types of elements give considerably accurate results even with small number of elements. Especially linear-element BEM is excellent.

Next is to examine how difference between outer element number and inner element number may influence the results. This problem may often appear when BEM are applied to turbine blades with arbitary section shape. Fig.5 shows some relations

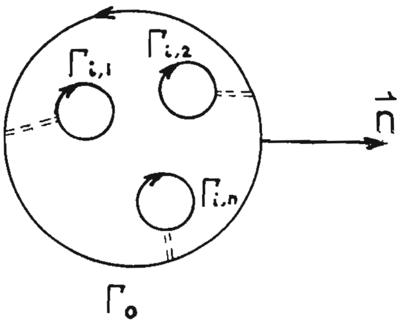


Fig.1 Multiple-connected domain

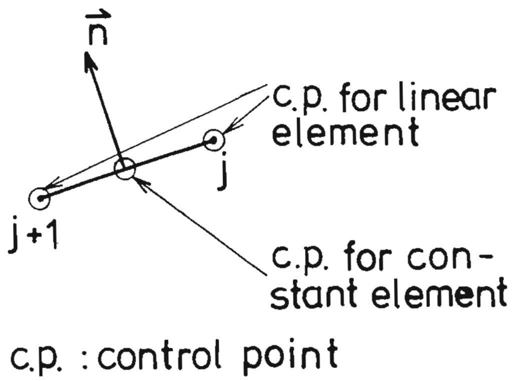


Fig.2 Difinition of a element

between relative errors and inner elements numbers N , with outer element number N fixed to be 100. In spite of considerable number of outer elements,obtained results seem to be less accurate than those of equi-division case mentioned above. It may be somewhat confusing to see that even in the case of same number of inner element number, the equi division case gives better results. Looking into the heat flux on the inner surface, however, unequi-division case turns out to give more accurate results than equi-division case. It should be noted that outer and inner surfaces are practically identified as regular polygons, not circles. Applying Green's identity eq (2) with u=1 and v= θ to those polygons, the following relation can be derived.

$$\frac{\partial\theta/\partial n,_i\, r}{\partial\theta/\partial n,_o\, r} = \frac{N_o \sin(\pi/N_o)}{N_i \sin(\pi/N_i)} \qquad (26)$$

It is obvious that if any numerical errors can be ignored, equi-division case(N_i=N_o) would makes the left-hand side of eq.(26) unity, which is the same of the case of circular boundaries. On the other hand, unequi-division case(N_i<<N_o) gives non-unity value. That seems to be the reason for relatively high accuracy of equi-division case. This may not be always the case when it is applied to arbitrary-shaped turbine blades, but the above discussion will do in checking the results of heat transfer coefficient distribution on blade surfaces.

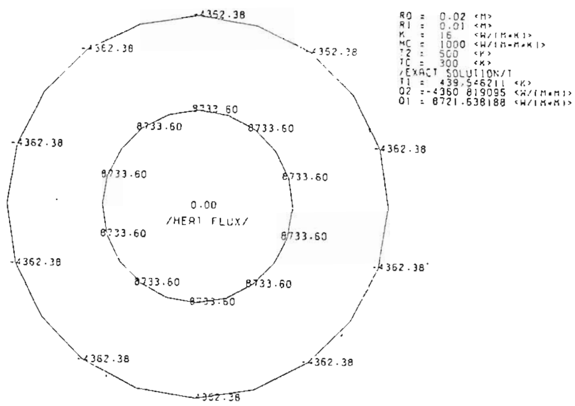


Fig.3 Test case -annular tube-

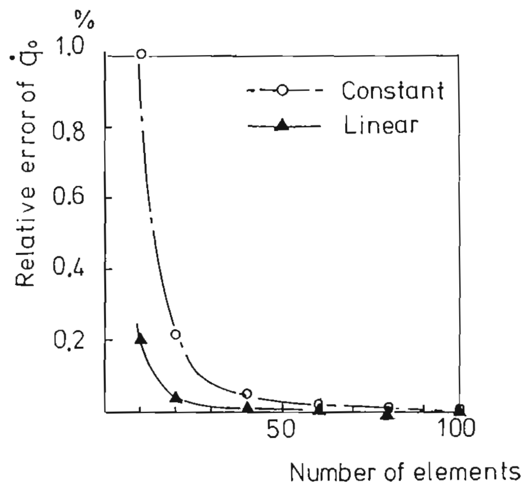


Fig.4 Relative error versus element numbers

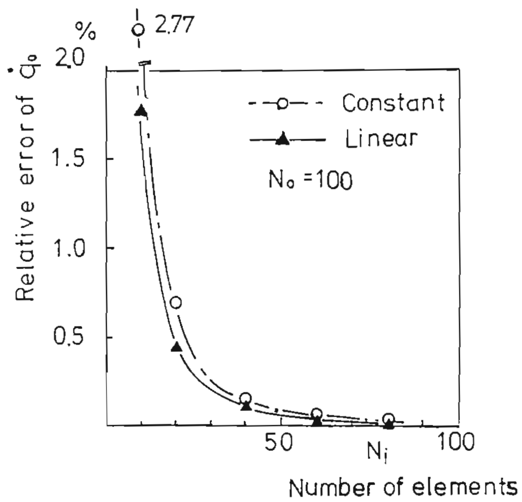


Fig.5 Effects of unequi-divided boundaries

4.1.2 Effect of non-uniformities

In the above discussion, elements configurations are uniform and boundary conditionsare also uniformly specified on outer and inner boundaries. However, this is a rather rare case in the application of BEM to actual turbine blades. Therefore, it is important to examine the effects of non-uniformities of elements and boundary conditions on the results, mainly heat flux.

(1) Boundary conditions

Using the same annular tube shown in Fig.3, effects of non-uniform specification of boundary condition are checked. For the sake of simplicity, periodic temperature distribution is given as follows:

$$\theta = \theta_c + \frac{q}{\lambda} A + \frac{q}{\lambda} B\cos\phi , \qquad (27)$$

where

$$A = r_2 \ln\frac{r_2}{r_1} + \frac{r_2}{h_c r_1}$$

$$B = \frac{(1+h_c r_1) r_2^3}{r_2^2 - r_1^2 + h_c r_1 (r_2^2 + r_1^2)} + \frac{1 - h_c r_1}{[(r_1^{-2} - r_2^{-2}) + h_c r_1 (r_1^{-2} + r_2^{-2})] r_2}$$

Solving the heat conduction equation analytically, it is easily derived that the above temperature distribution brings about heat flux on the outer boundary as[19]:

$$\partial\theta / \partial r = (q_1 \cos\phi + q_2)/\lambda . \qquad (28)$$

Fig.6 and Fig.7 respectively show the calculated heat flux distributions based on the analytical solution given by eq.(28), where $q_1 = 5\times10^4$, $q_2 = 2\times10^5$ and other parameters are the same as those in the preceding section. From these figures it is found out that both constant and linear elements give fairly good results.

(2) Element configurations

In this case as shown in Fig.8, elements[4n,4n+1](n=1,2,..) on the outer boundary differ from other elements in length. First of all, uniform temperature distribution(ϕ=0 in eq.(27)) are given on the outer surface and heat flux distributions obtained by constant- and linear-element BEM are respectively shown in Fig.9 and Fig.10 where the results are based on reference heat flux(ϕ=0 in eq.(28)). Comparison between them reveals that constant elements produce somewhat periodically scattered heat flux on the outer surface, while linear elements give almost constant (and correct) results.

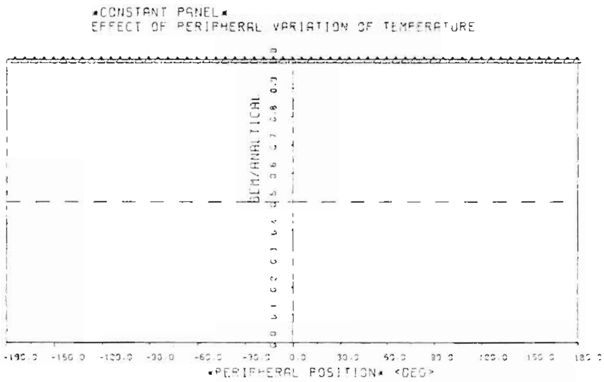


Fig.6 Effect of temperature variation

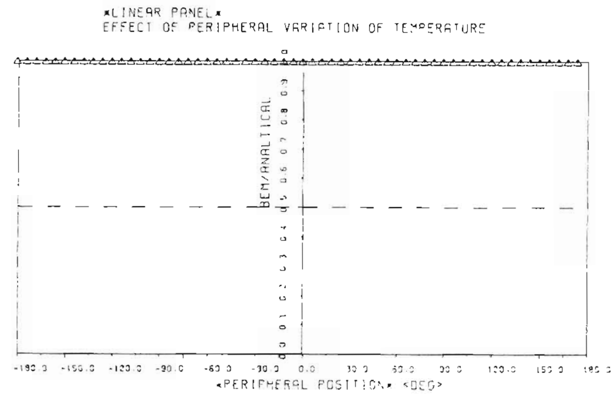


Fig.7 Effect of temperature variation

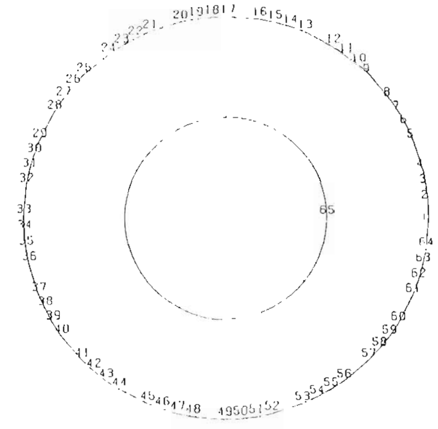


Fig.8 non-uniform division of outer boundary

Next periodic temperature distribution given in eq.(27) is used as the outer thermal boundary condition and it turns out that heat flux distribution by constant-element BEM presents considerable scattering as shown in Fig.11, where the results are based on the exact solution given in eq.(28). On the other hand, agreement between numerical solution by linear elements and analytical one are almost achieved with little deviations as can be seen in Fig.12. It should be noted that large deviations can be recognized at the place where temperature variation along the surface is also large, i.e. $\phi = \pm 90$deg.

As far as discussed in this section, it can be said that linear-element BEM seems to give better results than constant-element BEM. But due to the limited number of numerical experiments, such a thing cannot be generalized at once. Thus as a next step, application of the two types to a turbine blade will be made in the following section.

4.2 Calculations of Heat Transfer

Although the accuracy of the numerical codes was examined to some degree in the previous section, there's great necessity for checking their abilities to calculate heat transfer coefficients on actual turbine blades. Fortunately Hylton et al.[18] have conducted detailed experiments on temperature distributions around their turbine

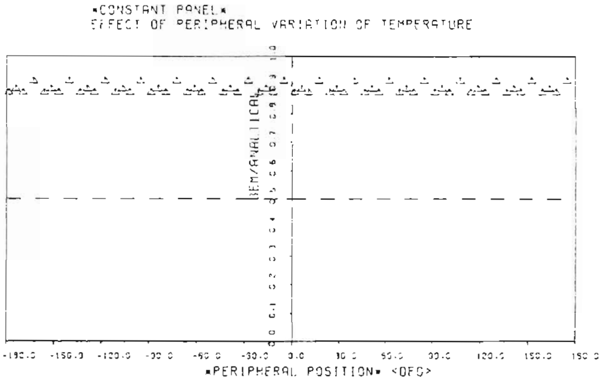


Fig.9 Effect of non-uniform division-constant-

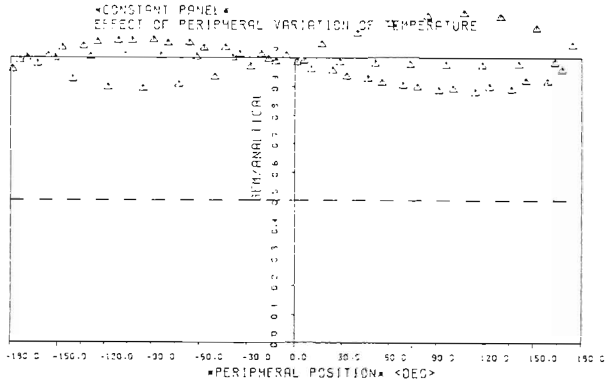


Fig.11 Effect of non-uniform division-constant-

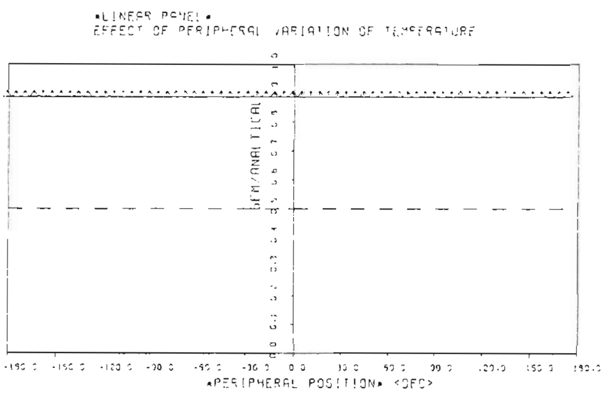


Fig.10 Effect of non-uniform division-linear-

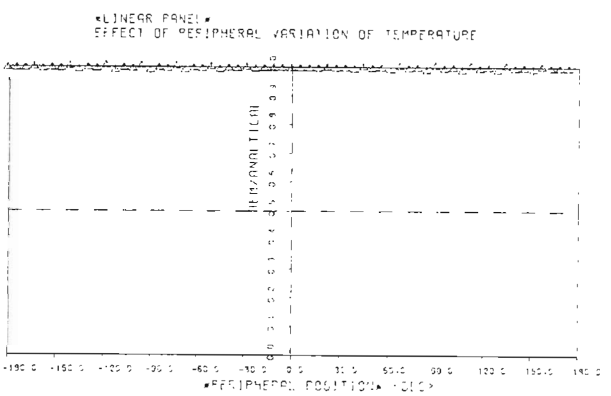


Fig.12 Effect of non-uniform division-linear-

nozzle vanes, and also calculated heat transfer coefficients by finite element method(FEM). Therefore it sounds reasonable to use their experimental data in order to test the performance of BEM proposed in this paper, and then see if the difference between the results by BEM and FEM can arise.

Fig.13 shows the turbine nozzle vane that was used by Hylton et al. in their experiments. (It was called C3X) As can be seen from Fig.13, that nozzle vane has 13 internal cooling cylindrical passages, where flow rates and temperatures of coolant were measured to determine inside heat transfer conditions. Temperature distributions on the nozzle vane were obtained by large number of thermocouples installed on the outer surface shown in Fig.14. Those informations are used as thermal boundary conditions of the numerical analysis as described before. Test conditions used in the following calculations are summarized in Table.1.

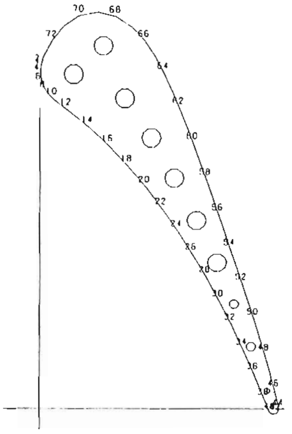
Code	$T_g(k)$	M_{in}	M_{out}	$Re_{,in}$	$T_u(\%)$
4412	796	0.17	0.9	$.52\times10^6$	6.5
5522	789	0.17	1.06	$.64\times10^6$	8.3

Table.1[18]

For the case of test code 4412, Fig.15 and Fig.16 respectively show heat transfer coefficients on the turbine blade surface calculated by constant- and linear-element BEM, where the results by FEM of Hylton et al. are also shown. Although thermocouples must have been installed near the trailing edge, several temperature data there are not given in their report[18]. Therefore temperature distributions at this region are defined by somewhat unreliable extrapolation in the BEM analysis. As can be imagined from rather rude treatment, large scattering of the results near trailing edge are clearly seen in both cases, while FEM analysis gives no results there. Such large scatterings of heat transfer, which are of course not true in a physical sense, also have their origin from the fact that heat flux $\partial\theta/\partial n$ changes drastically near the trailing edge where curvature of the blade surface changes significantly. This means that insufficient number of data around there may lead to severe deterioration of the results.

Ecxept for the trailing edge region, however, BEM, especially constant-element BEM and FEM produce almost same results. On the other hand, heat transfer distribution on the pressure surface calculated by linear-element BEM differs from that by FEM. These situations seem to be almost unchanged when number of elements on the blades and inner surfaces are doubled(156 and 20 elements respectively) as shown in Fig.17,18. Unfortunately it is not so clear about what causes such a difference between constant and linear elements. Although from the preceding arguments about the accuracy, linear -element BEM is found out to be superior to constant-element BEM, there arise great necessities for further investigations about them .

Fig.19 and Fig.20 show the heat transfer coefficients for the test code 5522. Also in this case, the result by constants elements almost agree with the result by FEM, while linear-element BEM shows some differences from FEM on the pressure surface. It should be noted that thermal conductivity of the blade material (ASTM 310 stainless steel) is estimated from the average of the surface temperature, though such a value used in the FEM analysis is not explicitly mentioned in the report[18]. This may be one of the reasons for the difference, and investigations are under way now.



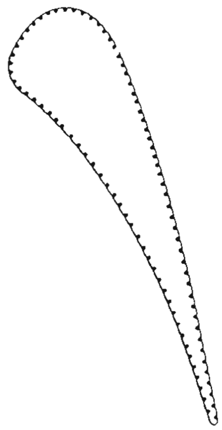


Fig.13 Turbine nozzle vane Fig.14 Thermocouples[18] (C3X)

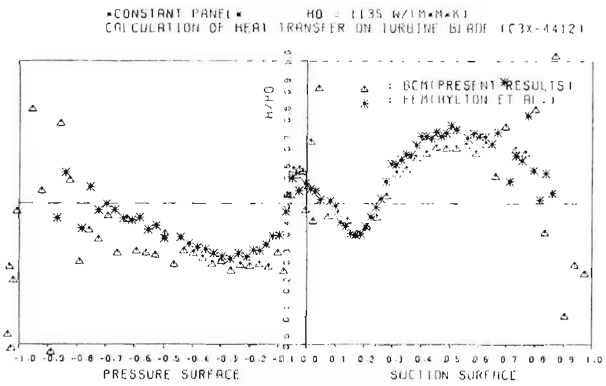


Fig.15 Heat transfer distribution-constant elements-(code:4412)

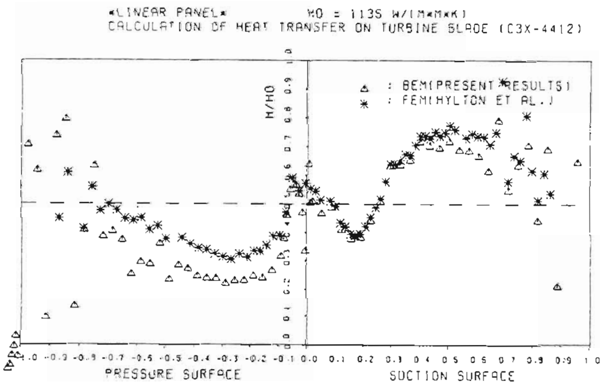


Fig.16 Heat transfer distribution-linear elements-(code:4412)

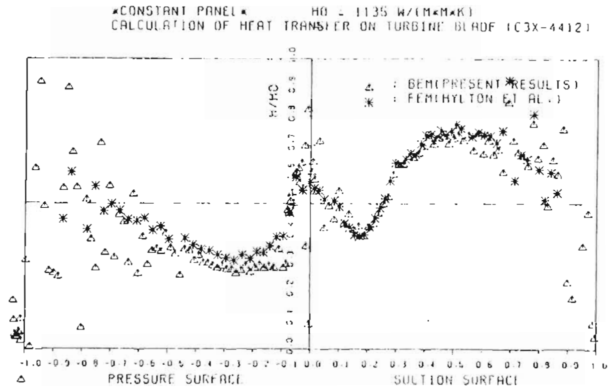


Fig.17 Heat transfer distribution-constant elements-(doubled element number:4412)

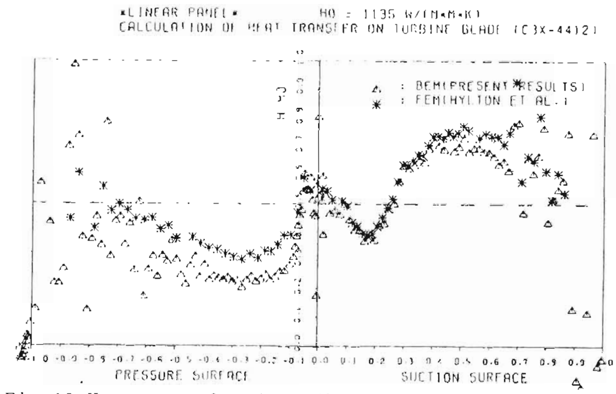


Fig.18 Heat transfer distribution-linear elements-(doubled element number:4412)

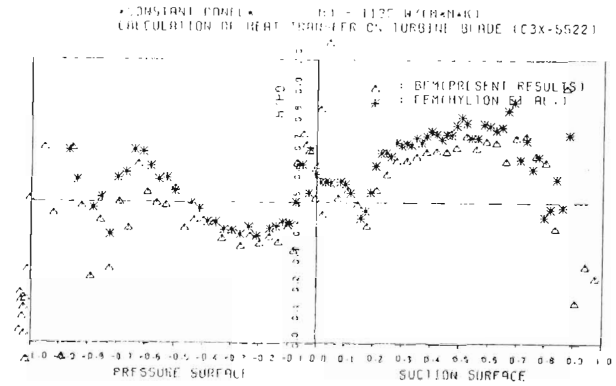


Fig.19 Heat transfer distribution-constant elements-(code:5522)

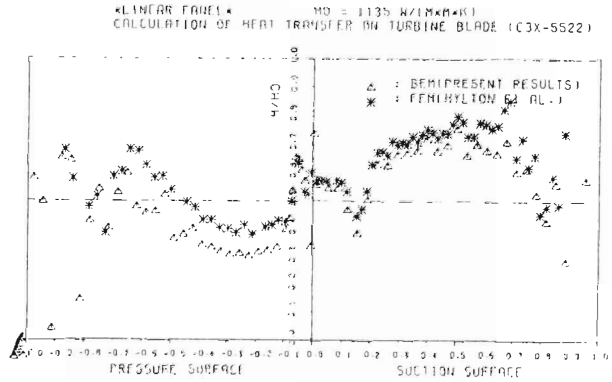


Fig.20 Heat transfer distribution-linear elements-(code:5522)6

5. Conclusions

This paper has proposed, as the first part of the consecutive studies, a numerical technique of Boundary Element Method to obtain heat transfer characteristics around a turbine blade surface using experimental data as the thermal boundary conditions.

Efforts are mainly focused on examining the numerical accuracy of BEM using two types of element for simple test cases and then actual turbine blade cases. It is found out that linear-element BEM seems to produce better results than constant-element BEM, but comparison between the results by BEM and FEM has revealed that there arises some difference between linear-element BEM and FEM by Hylton et al. Generalized conclusions cannot be derived from these limited investigationsand further numerical experiments are still going. In addition, measurements of heat transfer around turbine blades which have been developed by IHI, are also under way and results will be published soon.

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